International Baccalaureate

Exploring The Benefits of Tertiary Education

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Introduction:

As I reach the end of high school, I have been intrigued to look ahead at tertiary education[1] opportunities. I am fortunate to have access to a well-developed educational system, but I understand that this is not the case for many countries worldwide. Tertiary education, which is known as college where I live, can be extremely expensive. It is looked at as an investment, and I was interested in seeing if this investment was worth it. At first, I decided to do my research on the relationship between a country's gross domestic product[2] (GDP) and its tertiary education gross[3].

After a swift investigation, I found no relationship between a country's GDP and its tertiary education gross. The lack of relationship was due to countries with larger populations generally having a larger amount of money. It made sense that the more people a country has, the more money it would have. Instead of continuing my research around countries and their GDPs, I used their GDP per capita. GDP per capita is defined as the sum of gross value added by all resident producers in the economy plus any product taxes (fewer subsidies) not included in the valuation of output, divided by mid-year population (data.worldbank.org). Therefore, the aim of my exploration is to find a relationship between a country's GDP per capita and its tertiary growth.

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[1] Tertiary Education: the education level that follows secondary schooling or high school.
[2] Gross Domestic Product: the total monetary or market value of all the finished goods and services produced within a country's borders (investopedia.org).
[3] Tertiary education gross is defined as a value calculated by dividing the number of students enrolled in tertiary education regardless of age by the population of the age group which officially corresponds to tertiary education (databank.worldbank.org).

Research:

While I wanted to look at all 195 recognized countries, it seemed that it would be far too extraneous to collect data on all of them, so I started by taking a complete list of all countries and placing them into a random list generator. After running the random list generator, I took the first 100 countries listed. I chose to only collect 100 countries as it would be a large enough sample size to find a general relationship, but also not cause for a tedious and unnecessary amount of data collection. Then, I collected each of the 100 country's GDP per capita as reported by worldometers.info and their tertiary education growth as reported by data.wordlbank.org. The complete list of countries with their respective GDP per capita can be found in Appendix A.

While all of the GDP per capita data is up to date, unfortunately, the tertiary attendance rate for some countries is not. To keep my research accurate and legitimate, I decided to eliminate all countries that have not reported their data since 2017. Therefore, the data is from the 100 random countries excluding the countries listed in Appendix B.

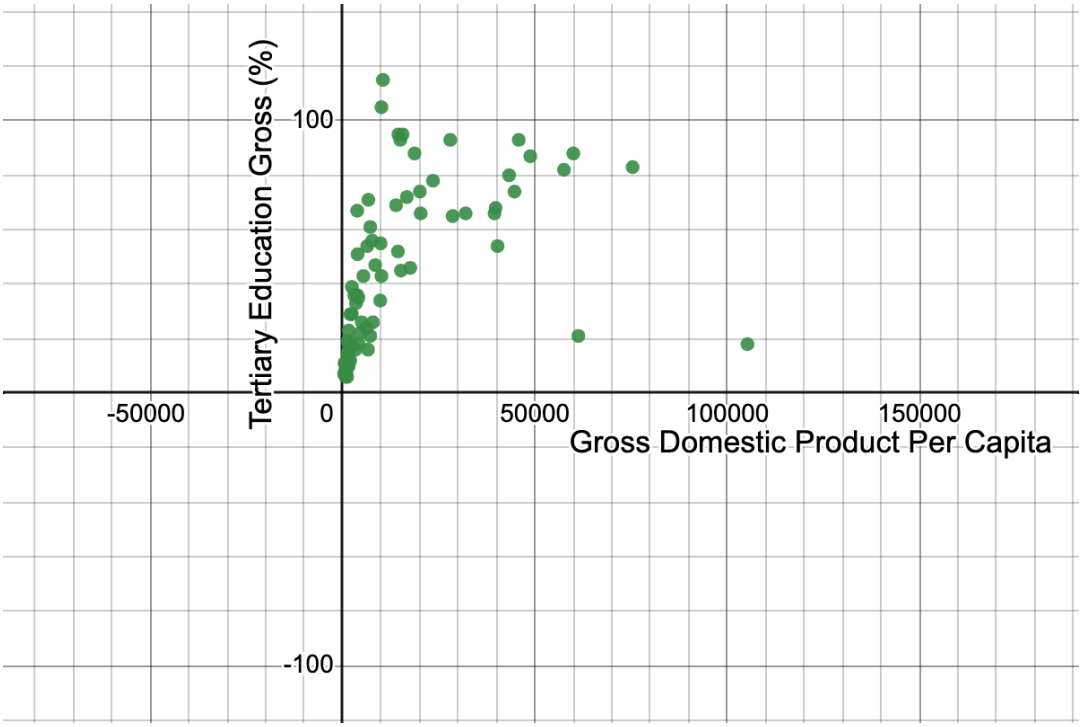
Two countries did not have any tertiary education gross reported; Kiribati and Bolivia.

After eliminating the previous countries, I was left with 74 countries with recent data. The next step for finding a relationship was to graphically represent my data using a scatter plot and create an equation to represent this data. The accuracy of the equation is determined by the R^2 value.[4]

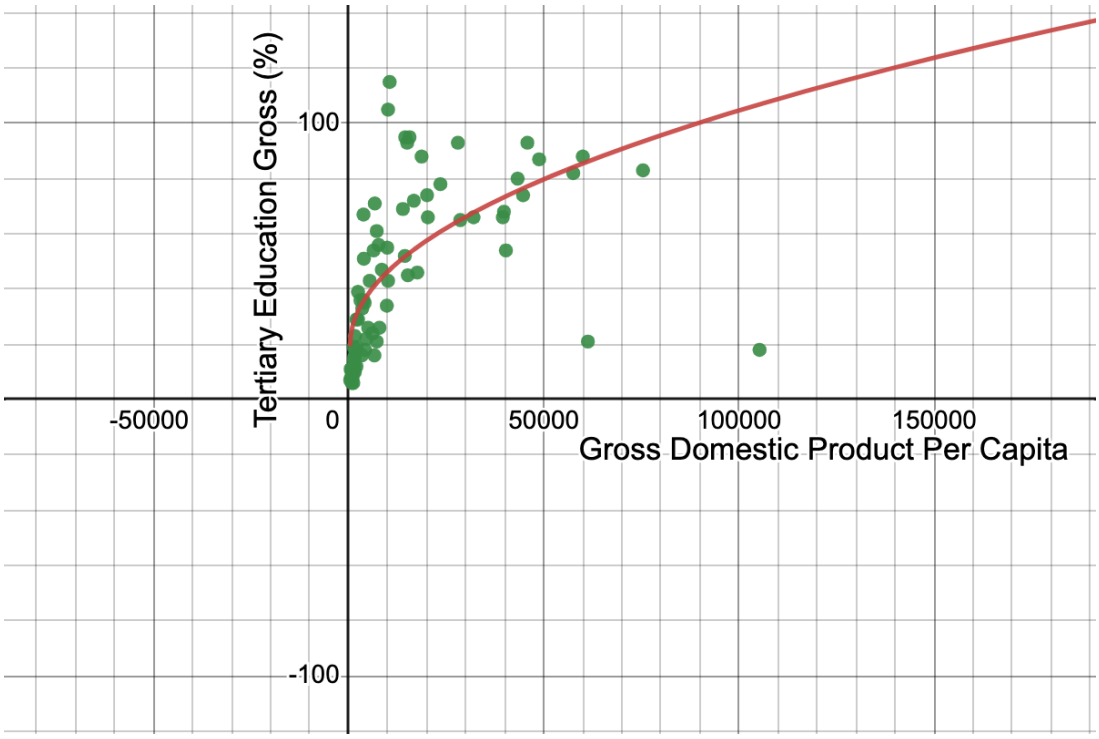
[4] R^2 Value: a statistical measure that represents the proportion of the variance for a dependent variable that's explained by an independent variable or variables in a regression model (investopedia.org).

Mathematical Application:

I chose to use the Desmos online graphing calculator to represent my data and equations, the y-value representing the tertiary education gross, and the x-value representing the GDP per capita. While finding the variables for my equations, I decided to round to 6 significant figures. Doing this allowed me to have very accurate numerical values without having values that stretched on for an extraneous amount of time.



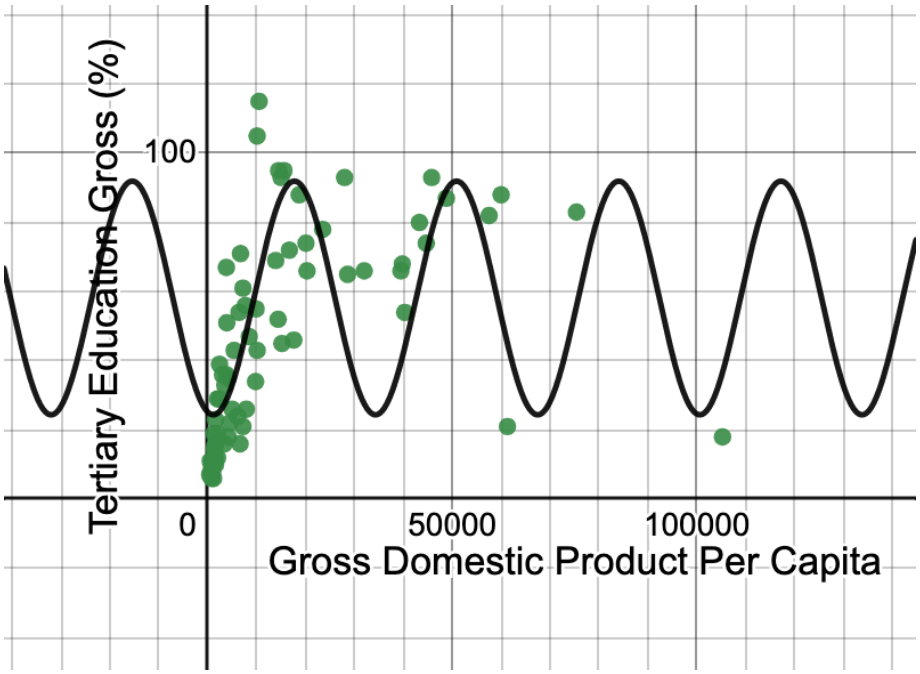
Graph with Equation 1:



Equation	Variables	R^2 Value
$y_1 \sim a\sqrt{x_1 - h} + k$	a = 0.268215, h = 400, k = 20.0306	0.3613

I first decided to try a square root equation as the data seemed to emulate the shape of one but quickly came to find that the R^2 value was fairly low.

Graph with Equation 2:

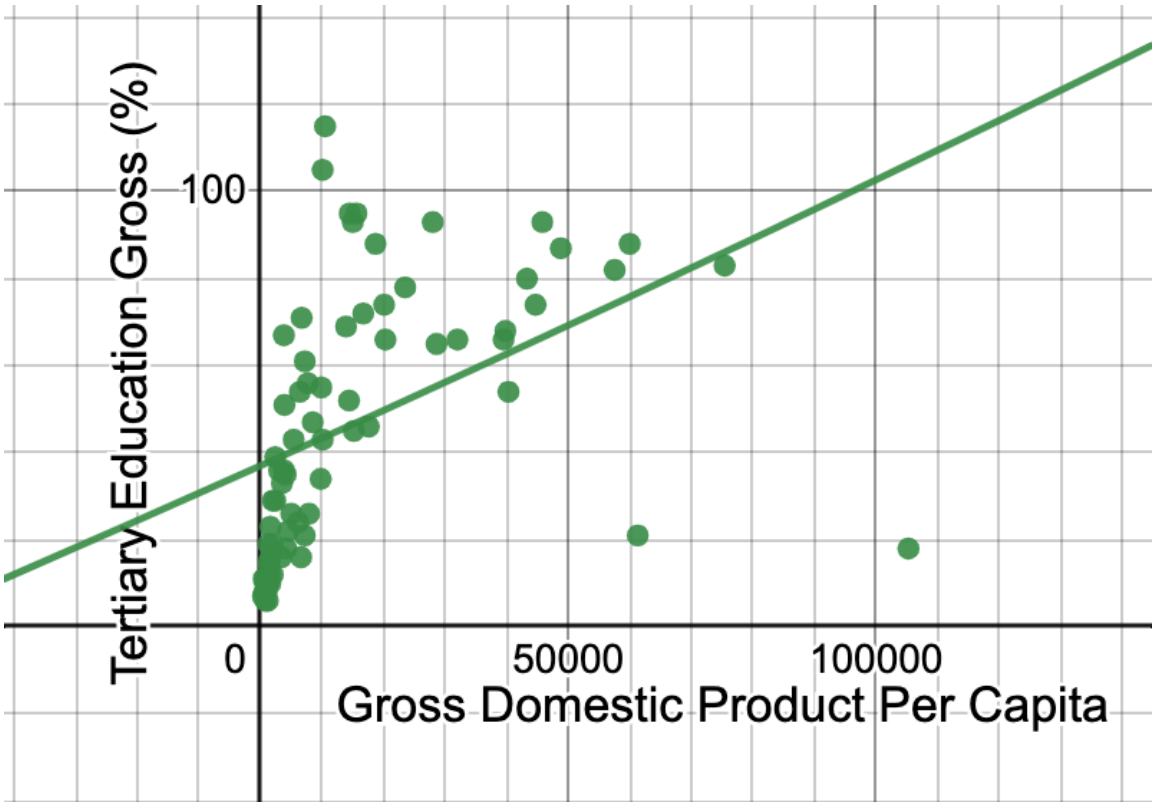


$y_1 \sim a\sin\left(bx_1 - h\right) + k$	a = 33.7853, b = 0.000189435, h = 1.78778, k = 58.1623	0.5487

Next, I tried to use a sinusoidal equation which gave me a much better R^2 Value. I thought that this one might work because, toward the end of the graph, you can see the points oscillate. Although this one gave me a better value, I felt that it was impacted too much by the influential points[5] and would be irrelevant once they were removed.

[5] Influential Point: a point that has a large impact on the regression (stat.cmu.edu).

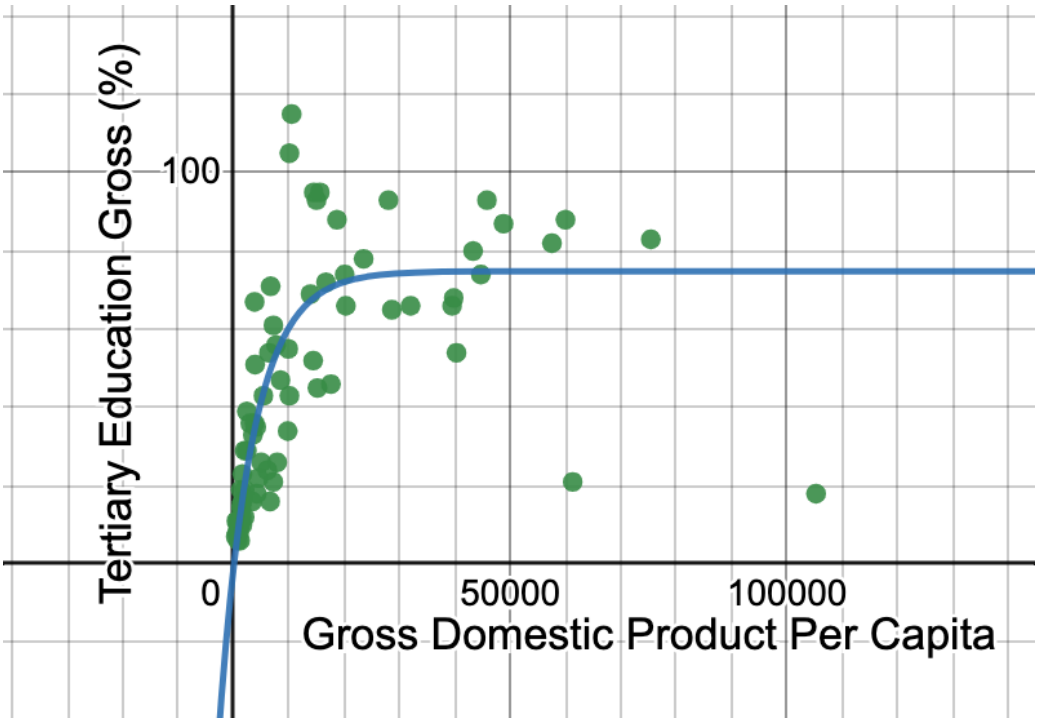
Graph with Equation 3:



$y_1 \sim a(x_1 - h)^2 + k$	a = 2.3327 * 10^{-10}, h = -1.3573 * 10^{6}, k = -392.733	0.1735

After the sinusoid, I was curious to see how a quadratic would work. Once it was graphed, it really ended up looking like a linear equation. I moved on from this one, as its relationship with my data was insufficient.

Graph with Equation 4:

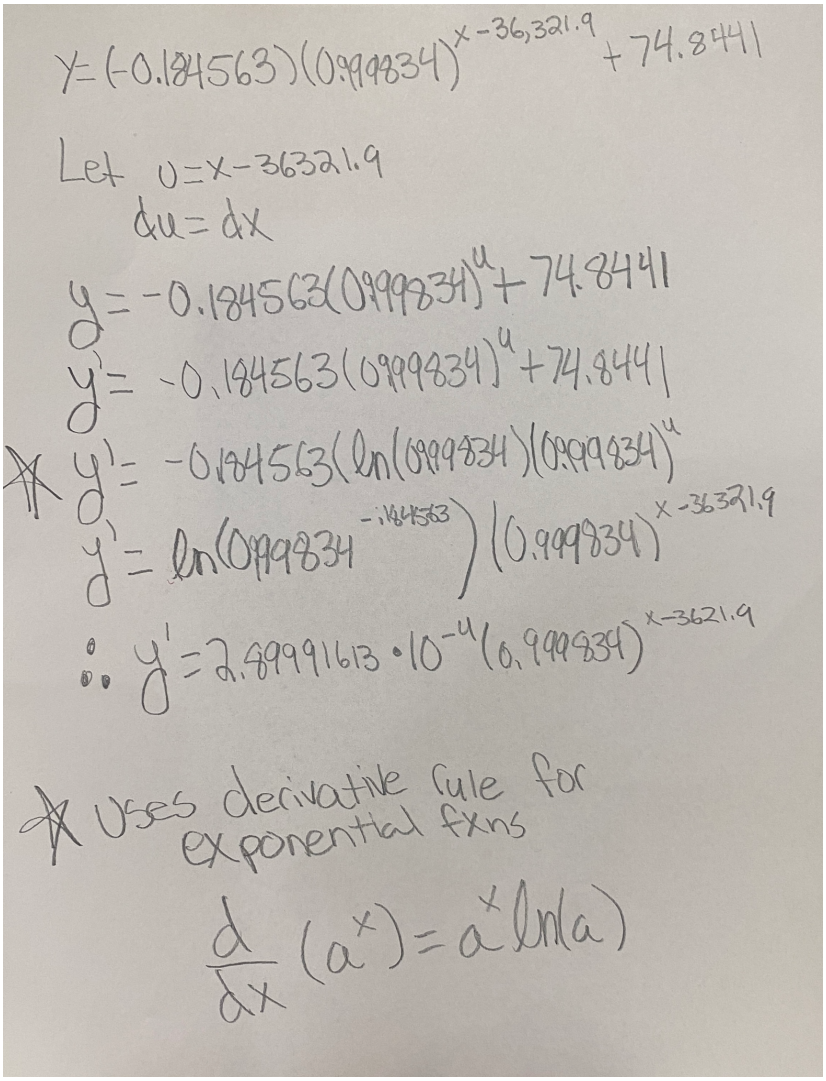


$y_1 \sim a \cdot b^{(x_1 - h)} + k$	a = -0.184563, b = 0.999834, h = 36321.9, k = 74.8441	0.5926

Finally, I chose to graph an exponential equation. I thought that it might work out pretty well since my data levels off, and, based on the numerical values, an exponential function could mimic this relationship. It worked out really well, providing me with the best R^2 value. I also knew that once the influential values were removed, I would be left with a much better relationship.

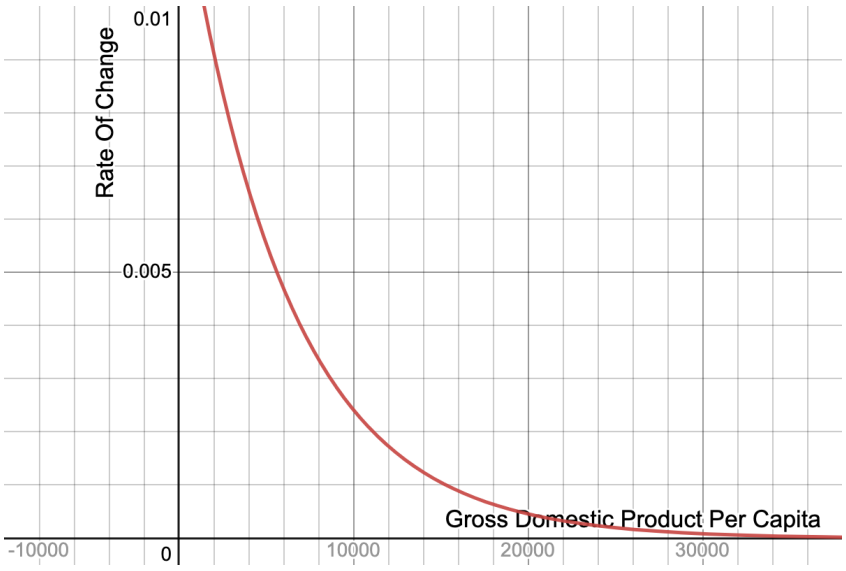
Mathematical Application 2:

Figuring out to what extent tertiary education gross influences gross domestic product per capita is an important step in my journey of figuring out if college is worth it. I thought that looking at the derivative of equation 4 (the best equation I came up with), would reveal if tertiary education gross continues to impact gross domestic product per capita at a constant rate, or if its impact gets smaller as it grows larger. I followed the chain rule, power rule, and exponential rule to find the derivative of equation 4:



Taking this derivative was something that needed more complex rules than I have learned in my math class before, as it needed to use the exponential rule, which was something I didn't have very much practice with. This was the most useful application of derivatives I have experienced since I was able to apply the principle that derivatives show the rate of change and use that in my model to see, in quantitative ways, how GDP per capita is affecting tertiary education over time.

Now, to understand the overall impact of tertiary education gross on the gross domestic product per capita, I can graph the derivative I found on desmos to see the relationship between the rate of change and the gross domestic product per capita. I decided to set my window to view the x-axis between -2000 and 30000 because around 30000 it starts to reach a horizontal asymptote. I decided to set the window for the y-axis to be between -.005 and 0.01 since the rate of change was incredibly small. It was small due to the fact that the tertiary education gross was all less than 150, while the gross domestic product per capita was measured at values up to 105280.



As seen in the graph, the equation is increasing at a decreasing rate. It is increasing the most between the values of 0 and 10000 and then stops increasing at around 30000. At 10000 gross domestic product per capita, the tertiary education gross is around 60%, while at 30000, it is approximately 74%. This means that if a country's tertiary gross is between 0 and 60% it has a far more drastic impact on its gross domestic product per capita than if it was 74% or greater. Tertiary education gross really only has an impact on the gross domestic product per capita to a certain extent, and does not infinitely influence it.

Mathematical Reasoning:

The equation used in graph 4 is confirmed as the strongest equation because it has the largest R^2 value. Generally, an R^2 value of 0.7 and above shows a strong relationship between data. As seen on the graph, some points could quite obviously be labeled as influential points that could be removed to create a stronger equation. It is the mathematician's task to determine what counts as an influential point. To keep my research accurate and legitimate, I have decided only to remove points that affect the R^2 value by 0.028 or greater. Points that meet this criterion have been removed including the following:

- Greece (19,200 GDP Per Capita, 149 Tertiary Growth)
- Luxembourg (105,280 GDP Per Capita, 18 Tertiary Growth)
- Qatar (61,300 GDP Per Capita, 21 Tertiary Growth)

These countries have reasoning for being so far off the standard relationship.

Greece seems to have a very low amount of wealth compared to its high tertiary growth, while Luxembourg and Qatar have very high wealth with low tertiary growth. I decided to research further potential reasons for this to occur.

Greece: According to the Organisation for Economic Co-operation and Development (OECD), Greece has one of the highest enrollment rates for both the age groups 20-24 and 40-64, and one of the highest average age of students enrolling into master programs. Since tertiary gross is measured by taking the total amount of people enrolled in tertiary education programs against the number of total people of the age for tertiary education and has such a high amount of tertiary students over the age of 40, it makes sense that the tertiary gross would be so high. According to Investopedia, Greece has a poor economy due to its government's spending habits, so the mismatch between tertiary gross and GDP per capita makes sense.

Luxembourg: According to USNews, Luxembourg has meager taxes, so it has enticed many large corporations to set up headquarters there. Due to its small population but big cooperation presence, Luxembourg has maintained an incredibly high GDP per capita. Although Luxembourg has a perceivably good education system, the general population has little motivation to take tertiary education opportunities because they already have the wealth to live comfortably.

Qatar: According to Business Insider, Qatar has become very rich due to the vast amount of oil found there. The oil industry dominates the country, and its GDP per capita remains high since it has such a low population. The country's economic system doesn't promote the pursuit

of education further than the secondary level, causing a gap between the tertiary gross and GDP per capita.

After the removal of these influential values, equation 4 maintains an R^2 value of 0.7105 and still remains the equation with the highest. When the variables of the equation are replaced by their values, the equation comes out to $y = (-0.520128)(0.999861)^{(x - 35991.4)} + 78.6204$.

As previously stated, an R^2 value of 0.7 or greater generally represents an equation with a strong relationship. This means that this equation proves a relationship both algebraically and graphically for the domain of my data. Therefore, there is a relationship between tertiary gross and the GDP per capita of a country. Because of this, I can make the inference that, generally, education promotes better wealth. After doing this research, I have found that exploring more tertiary education opportunities will likely benefit me in the future, as it seems that they promote an individual's overall wealth.

Extension:

To test the capabilities of the relationship I found, I decided to test my equation on the two countries I previously stated did not have a reported tertiary education gross (Kiribati and Bolivia) to try and predict their tertiary education gross. After plugging their GDP per capita into my equation, and rounding the final values to the nearest whole number, I can predict Kiribati's tertiary education gross should be 17% and Bolivia's tertiary education gross should be 30%.

Reflection:

I would have assumed that the Tertiary Education Gross of a country would have positively influenced its Gross Domestic Product Per Capita. It is commonly believed that college or tertiary education shapes us into being better for the workforce. Doing this research helped me understand to what extent college actually helps someone's financial future. Now,

rather than just assuming that college will benefit me, I can see that the initial investment of tuition will likely be worth it as it generally seems that the countries with the highest tertiary education gross have the highest gross domestic product.

Conclusion:

Overall, my research yielded strong results, and I plan to maintain interest in my predictions. I will likely check back on the reported tertiary education gross for countries to find out if I was able to predict the tertiary education gross for Kiribati and Bolivia accurately. It is also important to consider the countries that I chose to leave out due to outdated results. If I had included them in my data, I wouldn't have come up with modern results, but it is important to note that my data would likely be different if I was able to include them. I feel that my research has proved that tertiary education/college is generally a worthwhile investment, which has made me more comfortable with the idea of spending so much money as it seems to pay off. My research proved that better education leads to greater wealth and also increased my interest in tertiary education, and has greatly benefitted my math skills as a whole.

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Appendix A:

#	Country Name	GDP Per Capita (Rounded to nearest 100)	Tertiary Education Gross (%)	Year Tertiary Education Gross Reported
1	Cyprus	18700	88	2019
2	Greece	19200	149	2019
3	Oman	15200	45	2020
4	Saint Vincent and the Grenadines	7100	24	2015
5	Myanmar (Burma)	1300	19	2018
6	Kiribati	1600		N/A
7	Chile	15000	93	2019
8	Italy	32000	66	2019
9	Ethiopia	800	10	2018
10	Vietnam	2400	29	2019
11	The Gambia	700	3	2012
12	Armenia	3900	51	2020
13	United States	60000	88	2019
14	United Kingdom	39500	66	2019
15	Turkmenistan	6600	16	2020
16	Belize	5000	26	2020
17	United Arab Emirates	40300	54	2020

18	Gabon	7200	21	2019
19	Cameroon	1400	14	2018
20	Belgium	43300	80	2019
21	Spain	28000	93	2019
22	Poland	13900	69	2019
23	Haiti	800	1	1986
24	Uzbekistan	1600	16	2020
25	Nigeria	2000	12	2018
26	Peru	6700	71	2017
27	Egypt	2400	39	2018
28	Lesotho	1200	10	2018
29	Bhutan	3400	16	2020
30	Grenada	10100	105	2018
31	Denmark	57545	82	2019
32	Estonia	20100	74	2019
33	Dominican Republic	7200	61	2019
34	Malta	28600	65	2019
35	Slovakia	17600	46	2019
36	Mozambique	400	7	2018
37	Cambodia	1400	15	2019
38	Mauritania	1200	6	2020
39	Indonesia	3800	36	2018

40	Thailand	6600	49	2016
41	Norway	75428	83	2019
42	Libya	5800	60	2003
43	Rwanda	800	6	2019
44	Botswana	7900	26	2020
45	Czech Republic	20300	66	2019
46	Congo Democratic Republic of the	500	7	2016
47	Sierra Leone	500	2	2002
48	Pakistan	1500	12	2019
49	Qatar	61300	21	2020
50	Japan	38200	46	1999
51	Panama	15200	48	2016
52	Montenegro	7700	56	2020
53	Luxembourg	105280	18	2019
54	The Bahamas	31900	15	1995
55	Guatemala	4500	22	2019
56	North Macedonia	5400	43	2018
57	Kenya	1600	10	2019
58	Zambia	1500	4	2012
59	Tunisia	3500	33	2020
60	Maldives	9800	34	2019
61	Germany	44700	74	2019

62	Argentina	14500	95	2019
63	South Africa	6100	24	2019
64	Comoros	1300	9	2014
65	Hungary	14400	52	2019
66	India	2000	29	2020
67	Bangladesh	1600	23	2020
68	Turkey	10500	115	2019
69	Eswatini	3900	7	2013
70	Brazil	9900	55	2019
71	Azerbaijan	4100	35	2020
72	Philippines	3000	36	2021
73	Colombia	6400	54	2020
74	Bolivia	3400		N/A
75	Benin	800	11	2020
76	Malaysia	10100	43	2020
77	Finland	45800	93	2019
78	Chad	700	3	2015
79	Trinidad and Tobago	16000	12	2004
80	Antigua and Barbuda	15800	25	2012
81	Palau	16300	55	2013
82	Liberia	700	12	2012

83	Tonga	4200	18	2020
84	Papua New Guinea	2400	2	1999
85	Ghana	2000	19	2020
86	Lithuania	16700	72	2019
87	Latvia	15600	95	2019
88	Lebanon	7900	26	1985
89	Netherlands	48800	87	2018
90	Dominica	7000	7	1993
91	Cuba	8500	47	2020
92	Paraguay	5800	35	2010
93	Yemen	1100	10	2011
94	Tanzania	1000	8	2020
95	Slovenia	23500	78	2019
96	France	39800	68	2019
97	Suriname	5300	13	2002
98	Afghanistan	500	11	2020
99	Burkina Faso	600	8	2020
100	Georgia	3800	67	2020

Appendix B:

- Saint Vincent and the Grenadines (2015)
- The Gambia (2012)
- Haiti (1986)
- Thailand (2016)
- Libya (2003)
- Democratic Republic of the Congo (2016)
- Sierra Leone (2002)
- Japan (1999)
- Panama (20160
- The Bahamas (1995)
- Zambia (2012)
- Comoros (2014)
- Eswatini (2013)
- Chad (2015)
- Trinidad and Tobago (2004)
- Antigua and Barbuda (2012)
- Palau (2013)
- Liberia (2012)
- Papua New Guinea (1999)
- Lebanon (1985)
- Dominica (1993)
- Paraguay (2010)

- Yemen (2011)
- Suriname (2002)